ON EXTREME VALUE THEORY IN MODELING NIGERIA MARINE AND AVIATION INSURANCE CLASS OF BUSINESS

Oyewole O[1], Adesina O.S[2], Adekola L.O[3]
[1,3]Department of Physical Sciences, Bells University of Technology, Ota, Ogun State, Nigeria
[2]Department of Mathematical Sciences, Olabisi Onabanjo University, Ogun State, Nigeria
Email of corresponding author: olumideststats@gmail.com

Abstract: *Extreme value theory is applied to model extreme occurrences, and it is applied in business and finance to measure tail risk. Marine and aviation class of insurance business is an important component of non-life business because it insures all risks relating to aviation and marine vessels which play a significant role in the economic development of Nigeria. Recent claims experience has necessitated the investigation of tail risks with small probability of occurrence but with high potential impact on company's survival. The study employs the Extreme Value Theory (EVT) to estimate the minimum expected claims for the marine and aviation insurance business using historical claims data. Diagnostics plot like the mean excess plot suggest the threshold to chosen, to fit Generalized Pareto model based on EVT and the excess distributions were obtained over a chosen threshold. Linear Q-Q plots and tail plots reveal that parametric model fits the data well. VaR estimate was finally obtained using the extreme value method at 5% confidence interval and the empirical results show that Extreme VaR is most suitable to calculate VaR as against the Historical and Gaussian methods. This will guide proper underwriting process and loss reserving in this class of business.*

Keywords: *Extreme Value Theory, Insurance Claims, Marine and Aviation Insurance, Tail risks, Generalized Pareto Distribution*

1. INTRODUCTION

Marine and Aviation transport system is very critical in the development of world economy, without which international trade will be highly impaired. Records show that about a significant per cent of global trade by volume and value is carried by maritime facilities with the figures higher in developing countries (Thana, 2013). In addition, the aviation transport has brought about improvement in world trade accomplishments by enabling faster and easier movement of goods and passengers as well as providing millions of jobs. Aviation provides jobs for several millions of people and contributes up to about $2.4 trillion US Dollars in GDP, and it anticipated increasing to $6 trillion in GDP by next two decades (Cederholm, 2014). In Nigeria, the maritime industry is a critical sector in the economy which contributes immensely to transportation, facilitates trade and commerce, generates huge revenue, promotes tourism, provides job opportunities, ensures socio-political harmony, enhances defense and security especially in the area of territorial protection, and develops other economic activities (Igbokwe, 2001).

It is expected that such critical assets of the marine and aviation sector need insurance protection in the case of property damage and potential liabilities. Generally, insurance industry play significant roles in economic development of a nation in diverse ways especially in providing the needed funds for the replacement of lost or damaged property (Shittu, 1998). It also serves as security for loans; banks sometimes insist that the loan given to borrower be insured so as to be able to recover their money in case the borrower defaults. It also helps to stimulate individual's business, particularly in case of sudden death. For the marine and aviation sectors, insurers provide covers against risks of loss or damage to marine vessels and aviation hulls, marine and air cargoes, freights, collision and passengers' liability. Study by (Swiss Re, 2013) shows that global Marine and Aviation insurance has increased tremendously over the past decade which premiums have been estimated to be 44 billion US Dollars as at 2012

In Nigeria the context, (Onuoha, 2015) showed how a five year gross premium income for marine and aviation insurance obtained from the Nigerian Insurers Association (NIA) increased significantly from 2009 to 2013. Just as large pools and huge premiums are required in marine and aviation insurance, specialized technical skills are also a prerequisite for profitable underwriting of this class of business. In many instances, poor underwriting practices as reflected in premium rates coupled with uncooperative attitudes of some airline operators constitute a major challenge (Baker, 2013). Baker (2013) further notes that if there is air crash for instance, there is a difficulty in confirming the actual passengers on board. The various challenges confronting the insurers in the Nigerian insurance market have prompted many domestic airlines to seek insurance cover from foreign underwriters while at the same time trying to comply with the local content law (Akah, 2014).

About 80 per cent of world trade currently travels by sea, signifying around ninety three thousand (93,000) merchants' vessels, 1.25 million seafarers, and almost six billion tons of cargo. In recent years, the world shipping sector is faced with piracy which is one of the oldest crimes around the world against the trade (Katides, 2013). This is in addition to major losses in air crashes, stranding and sinking of marine vessels and burgeoning passenger liability claims. A major difference between marine insurance and other forms of insurance is that it involves consignments or vessels that are making ports of call throughout the world, so, different cultures, infrastructures and laws are involved and thus making the subject of pricing for marine insurance complex even for the actuarial profession (Nieh & Jiang, 2006).

The risk of large losses and consequently large insurance claims can be modeled with Pareto, Gamma, and Lognormal distributions for deciding on deductible and premium levels as outlined by Nigmet, El-Habashi and Hamdy (1987). Consequently, statistical models have been established to capture and measure risk accordingly, some of which are adopted in the in this study. The Extreme Value Theory (EVT) approach using Generalized Pareto Distribution can be used to estimate the Value-at Risk (VaR) among other methods used in estimating VAR, where VAR measures the minimal anticipated loss over a period, with a given probability (Rufino & Guia, 2011). One of the advantages of estimating VaR using GPD technique is that the technique estimates VaR outside the sampling interval. Extreme Value Theory techniques are useful in solving for very high quantiles, which are useful for predicting crashes and extreme-loss circumstances. Rufino & Guia (2011) outlined three types of VaR, namely, (i) Parametric or Gaussian distribution (ii) Non-parametric or historical sample, and (iii) semi-parametric, which is the application of extreme value theory to model data. It was outlined that financial data are hardly normally distributed. In this study, techniques for estimating Value-at-Risk of insurance is adopted and compared. Extreme Value Theory shall be employed to estimate the minimum expected claims for the marine and aviation insurance business using historical claims data. The suitability of the extreme VaR modeled by Generalized Pareto distribution, Gaussian (Normal distribution) and Historical approach shall be investigated. The remainder of this article is arranged as follows: Overview of maritime and aviation industry, methods, material and data, explorative plots, results, implication of the study, and conclusion and recommendations.

2. Methods

The model used in the study is generalized Pareto distribution. The cumulative density function (cdf) of a two-parameter GPD distribution is as follows:

$$G_{\xi,\beta} = 1 - \left(1 + \xi \frac{x}{\beta}\right)^{\frac{-1}{\xi}}, \xi \neq 0 \qquad (1)$$

$$G_{\xi,\beta} = 1 - \exp(-x/\beta), \xi = 0$$

Where $\beta > 0, x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\beta/\xi$ when $\xi < 0$

ξ is the essential shape parameter of the distribution and β is an additional scaling parameter. If $\xi > 0$, the $G_{\xi,\beta}$ is re-parameterized form of the ordinary Pareto distribution used for large losses, $\xi = 0$ relates to the exponential distribution and $\xi < 0$ is known as a Pareto of type II distribution.

The case where $\xi > 0$ is the most relevant for risk management. Whereas normal distribution has moments of all orders, a heavy-tailed distribution does not possess a complete set of moments. Fitting a GPD with $\xi > 0$, we observe that $E(X^k)$ is infinite for $k \geq 1/\xi$ when $\xi = 1/2$, the GPD is a second moment distribution, when $\xi = 1/4$, the GPD has an infinite fourth moment . However, certain types of large claims data in insurance typically suggest an infinite second moment. From Classical Moment Based Estimator for First and Second Moment, it is observed that due to the fat tail of the GPD for sufficiently large scale parameter ξ, the r^{th} moments of the GPD only exist for $\xi < 1/r$ (Neslehova, Chavez-Demoulin., & Embretch, 2006).

2.1 Maximum Likelihood Estimation for GPD

With $G_{\xi,\beta}$ for the density of the GPD, the log-likelihood may be calculated to be:

$$L(\xi,\beta;Y_j) = -N_u In\beta + \left(\frac{1}{\xi} - 1\right)\sum_{j=1}^{N_u}\left(1 - \xi\frac{Y_j}{\beta}\right), \xi \neq 0 \quad (5)$$

$$L(\xi,\beta;Y_j) = -N_u In\beta - \frac{1}{\beta}\sum_{j=1}^{N_u} Y_j,\ \xi = 0 \qquad (2)$$

Further working suggested by Scott (1993) reveals that:

$$\hat{\xi}_{MLE} = -\left(\frac{1}{N_u}\right)\sum_{j=1}^{N_u} In\left(1 - \hat{\theta}_{MLE} Y_j\right)$$

and $\hat{\beta}_{MLE} = \dfrac{\hat{\xi}_{MLE}}{\hat{\theta}_{MLE}}$ (3)

2.2 Distribution of Exceedances

The interest is in modeling the tails of loss severity, and the approach for distribution of exceedances to be considered is the peak over threshold (POT) method. Considering an unknown distribution function F of a random variable X, the interest is estimating the distribution function F_u of values of x above certain threshold u.

The distribution F_u is called the conditional excess distribution function and is defined as:

$$F_u(y) = P(X - u \leq y \mid X > u), \quad 0 \leq y \leq x_F - u \qquad (4)$$

Where X is a random variable, u is a given threshold, $y = x - u$ are the right endpoint of F. We verify that F_u can be terms as F,

i.e. $$F_u(y) = \frac{F(u+y) - F(u)}{1 - F(u)} = \frac{F(x) - F(u)}{1 - F(u)} \qquad (5)$$

$0 \leq X \leq u$

from GPD, the tail index ξ gives an indication of heaviness of the tail, the larger ξ, the heavier the tail. Only distributions with tail parameter $\xi \geq 0$ are appropriate to model financial claims.

2.3 Value-at-Risk

Value at Risk is a threshold loss value, it is the capital sufficient to cover, in most instances, loss from a portfolio over a holding period of a fixed number of days, weeks, months as the case may be. The Value at Risk of a random variable X with continuous distribution function F which models losses on a certain financial instrument over a certain time horizon is given by

$$VaR_p = F^{-1}(1-p)$$

Which is outlined in details by (Adeleke *et. al.,* 2015 and Adesina *et. al*., 2016) and summarized in equation (8)-(11). Where VaR_p is the p^{th} quantile of the distribution F. Where F^{-1} quantile is the function, and is the inverse of the distribution function F. The VaR_p, tail distribution of a GPD which is defined as a function of GPD parameters. Expected shortfall is defined as the excess of a loss that exceeds VaR_p. Bringing out $F(x)$ from (5), we have:

$$F(x) = (1 - F(u))F_u(y) + F(u)$$

And replacing F_u by the GPD and $F(u)$ by the estimate $\left(n - N_u \big/ n \right)$, where n is the total number of observations and N_u the number of observations above the threshold u, we obtain.

$$\hat{F}(x) = \frac{N_u}{n}(1 - (1 + \frac{\hat{\xi}}{\hat{\beta}}(x-u)^{-\frac{1}{\hat{\xi}}} + \left(1 - \frac{N_u}{n}\right) \qquad (6)$$

Simplifying we have:

$$\hat{F}(x) = 1 - \frac{N_u}{n}((1 + \frac{\hat{\xi}}{\hat{\beta}}(x-u)^{-\frac{1}{\hat{\xi}}} \qquad (7)$$

Inverting (6) for a given probability

$$V\hat{a}R_p = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}p\right)^{\hat{\xi}} - 1\right) \qquad (8)$$

Expected shortfall can be written as:

$$ES_p = V\hat{a}R_p + E(X - V\hat{a}R_p \mid X > V\hat{a}R_p) \qquad (9)$$

The second term on the right is the expected value of the exceedances over VaR_p. The excess function for the GPD with parameter $\xi < 1$ is:

$$e(w) = E(X - w \mid X > w) = \frac{\sigma + \xi w}{1-\xi},\ \sigma + \xi w > 0 \qquad (10)$$

The function gives average of the excesses of X over varying values of threshold w. Similarly, from (4), for $z = VaR_p - u$ and X representing the excesses y over u we obtain

$$\hat{E}S_p = V\hat{a}R_p + \frac{\hat{\sigma} + \hat{\xi}(V\hat{a}R_p - u)}{1-\hat{\xi}} = \frac{V\hat{a}R_p}{1-\hat{\xi}} + \frac{\hat{\sigma} - \hat{\xi}}{1-\hat{\xi}} \qquad (11)$$

The computational formulas for the three approaches for calculating VaR as outlined by Rufino and Guia (2011) are as follows:

(i) Gaussian VaR

$$VaR(\alpha) = u + (N^{-1}(1-\alpha))\beta \qquad (12)$$

(ii) Historical VaR

$VaR(\alpha) = ((1-\alpha)\times N)^{th}$ observation of historical sample (13)

(iii) Extreme VaR

$$V\hat{a}R_p(\alpha) = u + \frac{\hat{\beta}}{\hat{\xi}}\left(\left(\frac{n}{N_u}p\right)^{\hat{\xi}} - 1\right) \qquad (14)$$

Where n= number of observations in the parent distributions, N_u = number of tail observations with parameters β and ξ substituted with the maximum likelihood estimate

2.4 Data Description

The data used are from the Nigerian Insurance Digest, a publication of the Nigerian Insurers Associations (NIA), where claims over a period of 2 years (2011 and 2012) were published. Data of four insurance companies for marine and aviation insurance class which occupies a single column in the publication were collected for the analysis and estimation. The data set was fitted into a GPD Model with maximum likelihood estimate. Software package by R Core team (2017) was used to carry out the analysis in this study, package fExtremes by Wuertz *et. al.,*(2013) was equally used to fit the GPD model. The presence of extreme values was tested in the data using the Grubb's test and quartile test and extreme values (outliers) were found in the data set. Also, to further examine the significance of the aviation and maritime industry to Nigeria economy, table 1 gives a descriptive statistics of gross domestic product (GDP), revenue and contribution of aviation industry to Nigeria GDP.

Table 1: Showing Descriptive Statistics of GDP, Revenue and Contribution of Aviation to GDP

Year	Descriptive	GDP, Current Prices, billion $US	Revenue in million naira	Contribution of Aviation Sector to GDP
1990-1996	mean	4.732	364.00	1156539.573
	SD	2.525	400.92	3059916.09
	Skew	0.834	0.834	0.00001
	Kurt	0.232	-1.244	-1.2000
1997-2004	mean	67.037	2700.125	248922559.17
	SD	29.136	1025.518	3469618.718
	Skew	1.093	0.227	0.0001
	Kurt	0.474	-1.128	-1.2000
2005-2012	mean	309.7375	6336.750	36223986.41
	SD	105.0602	1148.362	3469618.718
	Skew	0.165	-0.72	0.00001
	Kurt	-1.061	-1.939	-1.2000

Source: National Bureau of Statistics (NBS)

3. RESULTS

Table 2: Descriptive Statistics of Marine & Aviation Insurance Class of Business Claims

	Descriptive			
	MEAN	STDEV	SKEW	KURT
Leadway	9.68566	12.35721	3.992394	8.982724
Zentih	13.06045	18.94347	3.66514	13.28954
Mansard	10.71786	10.31947	1.749439	1.77732
Custodian	14.75716	12.17876	0.363282	-1.73202

Table 3: Grubb's test for extreme Value, H_A : There is an outlier in the data set

Company	G	U	p -value	Outlying Value
Leadway	3.1787	0.0165	5.613×10^{-13}	55.6414
Zenith	4.4421	0.1048	3.54×10^{-11}	97.2100
Mansard	2.8954	0.5355	0.009366	40.5972
Custodian	1.5964	0.5839	0.0366	34.1995

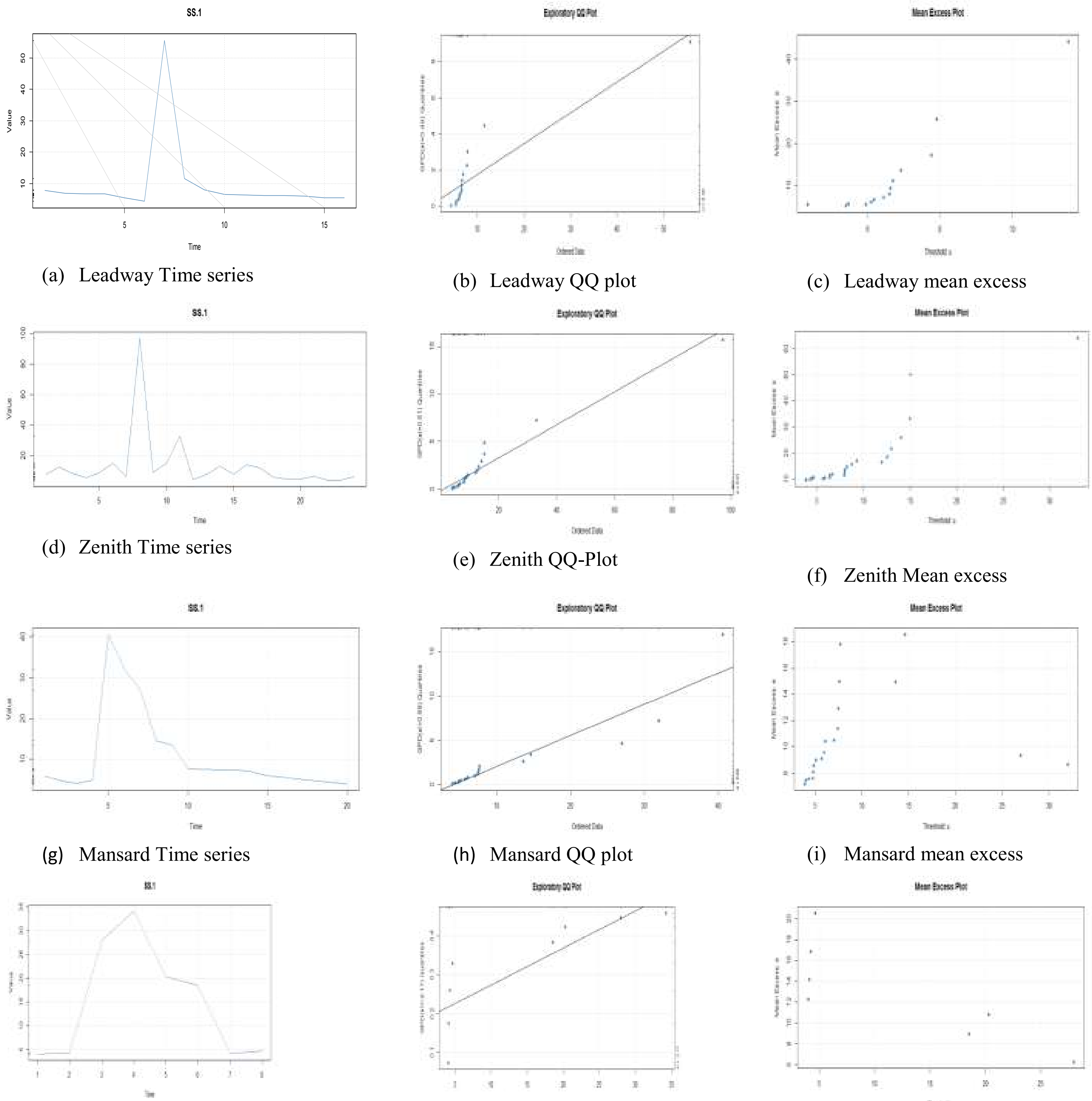
The above test statistics reveals that there are outlying value(s) in the each data set

Table 4: Normality tests of Marine and Aviation Claims (H_0 : Normal)

	Anderson Darling	Craver-Von Misses
Leadway	4.3285**	0.8625**
Zenith	4.7426**	0.9094**
Mansard	2.9237**	0.5632**
Custodian	0.6177**	0.1015**

** indicates significance at 1% $\mid p < 0.01$

An empirical analysis was carried out to model the marine and aviation insurance claims of the Nigerian insurance market. Four insurance companies' claims that were captured and modeled are discussed. The marine and aviation insurance claims data were fitted in GPD model and discussed.



(a) Leadway Time series
(b) Leadway QQ plot
(c) Leadway mean excess
(d) Zenith Time series
(e) Zenith QQ-Plot
(f) Zenith Mean excess
(g) Mansard Time series
(h) Mansard QQ plot
(i) Mansard mean excess
Custodian Time series
Custodian QQ plot
Custodian mean excess

Figure 1: Time series plots, Q-Q plots, Mean excess plots

Table 5: Value at risk, Threshold Selection and parameter of estimates

Value-at Risk estimates at α =5%				Threshold Selection and MLE Parameters		
Company	Historical	Gaussian	Extreme VaR	u	ξ	β
Leadway	-22.5	9.9	4.33823	4.5	0.48840	2.49874
Zenith	-30.2	17.5	3.83760	5.5	0.61182	4.09401
Mansard	-30.2	5.8	3.933945	5	0.69003	3.78689
Custodian	-32.0	3.9	4.03306	19	-2.1737	33.0397

With Threshold of 4.5, the shape of Leadway Issurance loss' data parameter ξ is greater than 0 implying a heavy tailed distribution. This can be interpreted to mean that the higher the value of the shape parameter, the higher the derived claim. The distribution for the excesses shows a smooth curve meaning GDP fit was a good fit for the data. The Value at Risk (VaR) with 5% level of confidence was ₦4.338233 million, which is a 1-in-20. For twenty four months period, it implies that the coming one month six day's loss for the company would exceed ₦4.338233 million. Similar results were obtained from the analysis of the Zenith Issurance loss data where the Value at Risk (VaR) with 5% level of confidence was ₦3.837603 million, also, a 1-in-20; and in Mansard Insurance loss data where the Value at Risk (VaR) with 5% level of confidence was ₦3.933945 million, which is also a 1-in-20.

With Threshold of 19, the shape of Custodian Insurance loss' data parameter ξ, however, is less than 0 implying a thin tailed distribution; hence Pareto type II is obtained. The distribution for the excesses shows a smooth curve meaning GDP fit was a good fit for the data. The Value at Risk (VaR) with 5% level of confidence was ₦4.0330636 million, which is a 1-in-20. For twenty four months period, it implies that the coming one month six day's loss for the company would exceed ₦ 3.0330636 million. Then precautions can be taken to mitigate against it.
From the study, we deduce that that tail risk can be measured accurately by adopting Value-at-Risk (VaR) initiated by extreme value theory and estimation being carried out using maximum likelihood estimate on Generalized Pareto distribution. Extreme VaR is found to outperform two other methods examined (Historical and Gaussian). It is imperative that proper capturing of tail risks will help in proper underwriting and claims reserving.

4. Conclusion and Recommendations

Detailed analysis is carried out to model marine and aviation industrial insurance losses of the Nigerian insurance companies. The presence of extreme values was tested using Grubb's test, and the data were found to contain extreme values. Findings in this paper reveal that Extreme Value Theory method of calculating VaR outperforms other methods of estimation as it is known for its ability to model the tail area of the distribution much better. Other methods of estimation perform well in high profile data, but extreme VaR are estimated irrespective of the number of data set.

Having estimated and compared the various ways of estimating value at risk, we make some recommendations as follows:

i. That extreme value theory approach should be adopted in estimating Value-at-Risk for insurance claims.
ii. Insurance companies should put measures in place to mitigate huge losses resulting from Aviation and Marine class of business.

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