EVALUATING THE ROBUSTNESS AND IMPLEMENTING DOUBLE ONE-SIDED HODRICK-PRESCOTT FOR CYCLE EXTRACTION

Jeerawadee Pumjaroen[1,*] and Yuthana Sethapramote[2]

Abstract

Cycle extraction is a crucial part of a business cycle analysis. This research aims to identify the robustness of cycle extraction methods, which indicates a business cycle's reliability. Interestingly, this research proposes the Double One-sided Hodrick-Prescott (DoneHP) for cycle extraction. This method was compared to four well-known methods, including the Hodrick-Prescott (HP), Double Hodrick-Prescott (DHP), Christiano-Fitzgerald (CF), and One-sided Hodrick-Prescott (OneHP) analyses. In addition to evaluating the revision of size and signal, the study also assessed the revision of the turning point.

Using four choices of Thailand's reference series, the results of the expanding window experiment showed the consensus that DoneHP was outstanding and had less revied cycle values, especially at the end of the cycle. Based on the overall ranking, the order of performance for the methods was DoneHP, OneHP, DHP, HP, and CF. The findings suggest DoneHP for cycle extraction in cases where the analysis is made in real-time and minimizing the revision of past estimation is preferred. In detail, DoneHP is recommended, followed by OneHP, HP, DHP, and CF when the priority is minimizing size revisions. However, the emphasis focuses on a steady signal and turning point, preferring as follows DoneHP, OneHP, DHP, HP, and CF.

Keyword: Cycle extraction, De-trending, Economic cycle, Hodrick-Prescott, Robustness

1. INTRODUCTION

The challenge for policymakers is to minimize fluctuations in the economy. Understanding the cyclical economic situation helps to build correct and efficient stabilization policies. One of the essential instruments for policymakers is business cycle analysis, which is used to identify the economic state and help identify measures to ensure economic stability, with the relevant information (Mazzi & Scocco, 2003; Padilla and Otero, 2022). Many researchers have applied the business cycle concept to identify leading indicators and develop economic early warning models (Gyomai & Guidetti, 2012; OECD, 2023; Pumjaroen, Vichitthamaros, & Sethapramote, 2020).

Various techniques have been proposed to study the economy's business cycle since the end of world war II. Burns and Mitchell (1946), who presented the business cycle concept, are usually referred to as the pioneers of the economic cycles in modern theory. There are many approaches for analyzing business cycles, such as the classical business

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cycle, the growth cycle, the growth rate cycle, and the recovery cycle. However, two approaches generally are applied to identify the fluctuations of the business cycle in the economic literature—the classical and the growth cycles (Mazzi & Scocco, 2003). These two cycles are different in their way of measuring the fluctuations of the economy. The classical cycle approach relies on trend-cycle data, while the growth cycle is based on de-trended data.

The study of Burns and Mitchell (1946) identified business cycles by the classical approach of trend-cycle data as sequences of expansions and contractions of the data representing the economy, such as the levels of total output. This approach has been dominant in the business cycle studies of the National Bureau of Economic Research (NBER) for many developed countries (Calderón & Fuentes, 2010; Mazzi & Scocco, 2003).

However, for countries with a high frequency of fluctuations, such as emerging countries, the classical approach might not be appropriate as in linear terms, this approach will hinder the extraction of cyclical phases (Gatfaoui & Girardin, 2015). The alternative approach, growth cycle, is based on de-trended data, and is most considered for this task (Padilla & Otero, 2022). The growth cycle concept proposed by Mintz (1969) keeps the same analysis framework of Burns and Mitchell (1946). However, it changes the definition of measuring aggregate economic activity to consider a deviation from the trend. The growth cycle approach therefore applies filtering techniques to extract the cyclical components from the time series data, making cycle extraction a crucial part of economic cycle analysis for this approach.

The results of an economic cycle are prone to change when new data is available to update the cycle analysis; all past economic cycle values will be revised. The discrepancy between the two results describes the same phenomenon called revision. In other words, revision refers to the historical economic cycle values that change when the newly available data for subsequent periods is added to the analysis. The magnitude of the revision can be a criterion to evaluate the cycle extraction methods in terms of robustness as a less relative revision reflects greater precision of the technique and its practical usefulness for economic and business applications. Even though the revisions probably occur from either the methodology or the updated source data (van de Minne, Francke, Geltner, & White, 2020), this research focuses on revisions from the cycle extraction methodology.

Various statistical techniques have been proposed to extract cyclical components. Hodrick-Prescott (HP) (R. Hodrick & Prescott, 1981; R. J. Hodrick & Prescott, 1997) is one of the most frequently applied for this task (Padilla & Otero, 2022). This method is considered the simplest variant of advanced filtering techniques (Benes & N'Diaye, 2004) since it requires the specification of only one parameter to optimize the smoothing trend, decomposing data into trend and cycle components.

Many studies, including the work of the OECD CLI system (OECD, 2023), have applied HP twice; this method is referred to as the Double Hodrick-Prescott (DHP) and increases the stability of the cyclical estimation (Gyomai & Guidetti, 2012; Pumjaroen et al., 2020).

However, HP's results have been criticized for generating end-point problems and spurious dynamic relations due to the dependency on using the full sample (Hamilton, 2018). Therefore, to avoid these problems, the One-sided Hodrick-Prescott (OneHP) method (R. J. Hodrick & Prescott, 1997) is optional for de-trending as the real-time version of the regular HD (Hamilton, 2018). Due to applying only a partial sample rather than taking all data as done in HP to estimate the trends and cyclical components, OneHP does not suffer from spurious predictability or end-point biases.

Owing to the criticism of using HP, some researchers have applied the Christiano-Fitzgerald filter (CF) (Christiano & Fitzgerald, 1999) to separate the trend and cyclical components (Pandey, Patnaik, &

Shah, 2017). CF is the de-trending and smoothing problem in the frequency domain. This method is a band-pass filter for a random walk time series based on a similar principle to the Baxter and King (BK) filter (Baxter & King, 1999). Gyomai and Guidetti (2012); Nilsson and Gyomai (2011) stated that CF outperformed on a long series over BK, converged in the long run to the optimal filter, and in real-time applications outperformed the BK filter.

A study of revisions in three methods, including the Phase Average Trend method (PAT), DHP, and CF of Nilsson and Gyomai (2011), indicated that DHP and CF outperformed PAT. However, the result was not a consensus on whether DHP or CF was outstanding. The researchers of this study pointed out that the DHP beats CF in signal stability but had a weaker performance in minimizing size revisions. Moreover, DHP could generate a problem regarding the criticism of Hamilton (2018) since it applied HP twice.

It is hoped that this study will pave the way for analyzing the economic cycle and identifying the leading indicators for Thailand, which is one of the emerging countries; hence the growth cycle approach is taken making the cycle extraction a crucial part. Regarding the problem of HP mentioned above, this study proposes using the Double One-sided Hodrick-Prescott (DoneHP) to extract the cyclical component. Instead of using HP twice as in DHP, DoneHP applies the OneHP two times to avoid the criticism of Hamilton (2018). Evaluating the performance of DoneHP focusing on robustness, the study compares this method to four famous cycle extraction methods, namely HP, DHP, CF, and OneHP; the preferred method should give less modification for each re-estimation. In addition to evaluating the revision of size and signal, accession of the turning point revision was also included in the study.

The succeeding sections of this paper are organized as follows. The next section briefly reviews the literature on cycle extraction methods. In the third section, the report describes the data and methodology of the research. The empirical results are shown in the fourth section. Finally, the conclusion and recommendations are given in the fifth section.

2. LITERATURE REVIEW

The literature review focused on four frequently used methods of cycle extraction, including Hodrick-Prescott (HP), Double Hodrick-Prescott (DHP), Christiano-Fitzgerald (CF), and One-sided Hodrick-Prescott (OneHP).

2.1 Hodrick-Prescott (HP)

Several statistical methods have been proposed for extracting the cyclical component of time series data. The Hodrick-Prescott method (HP), presented by Hodrick and Prescott (1981); and Hodrick and Prescott (1997), is one of the most commonly used methods for this task (Gyomai & Guidetti, 2012; Mazzi & Scocco, 2003; Nilsson & Gyomai, 2011; Wolf, Mokinski, & Schüler, 2020), especially for the study of regional business cycles in an emerging economy (Padilla & Otero, 2022). The HP filter is considered the simplest variant of the advanced filtering techniques (Benes & N'Diaye, 2004) that can be applied to any time series (Hodrick & Prescott, 1997).

The HP method decomposes time series data (y_t) into a trend (τ_t) and a cyclical (c_t) component as indicated by the following equations:

$$y_t = \tau_t + c_t;\ t = 1, 2, \ldots, T \tag{1}$$

$$y_{1:T} = (y_1, \ldots, y_T)' \tag{2}$$

$$\tau_{1:T} = (\tau_1, \ldots, \tau_T)' \tag{3}$$

$$c_{1:T} = (c_1, \ldots, c_T)' \tag{4}$$

HP aims to minimize the distance between the trend and the original data while minimizing the curvature of the trend series. The method requires only the smoothing parameter λ (λ>0) to optimize the trade-off between these two aims. The regular HP uses a full sample $(t = 1, 2, \ldots, T)$ to estimate the trend component $(\hat{\tau}_{t|T,\lambda})$, sometimes called a two-sided Hodrick-Prescott filter.

$$\left\{\hat{\tau}_{1|T,\lambda},\ldots,\hat{\tau}_{T|T,\lambda}\right\} = \underset{\tau_1,\ldots,\tau_T}{\arg\min}\left(\sum_{s=1}^{T}(y_s - \tau_s)^2 + \lambda\sum_{s=2}^{T-1}(\tau_{s+1} - 2\tau_s + \tau_{s-1})^2\right) \quad (5)$$

where λ is the smoothing parameter (λ>0) to estimate the trend component ($\hat{\tau}_{tT,\lambda}$). If the value of λ is higher, the extracted trend component will be smoother. The estimation of λ can be based on empirical observations—14,400 for monthly, 1,600 for quarterly, and 100 for annual data (Schlicht, 2005). The cyclical component will be estimated from $\hat{c}_{t|T,\lambda} = y_t - \hat{\tau}_{tT,\lambda}$

Even though HP is a popular filtering method for de-trending, there is criticism of the use HP regarding the generated results. Some critics indicate that HP's dependency on using the full sample results in spurious dynamic relations (spurious predictability). Additionally, there is an end-point problem, in that the filtered values have high differences between the sample's middle and end; the revision $\hat{\tau}_{tT,\lambda}$ and $\hat{c}_{t|T,\lambda}$ are made when new data is available (end-point bias) (Hamilton, 2018).

2.2 Double Hodrick-Prescott (DHP)

The Double Hodrick-Prescott (DHP) has been applied for cycle extraction in the OECD CLI system since 2008, replacing a combination of the Phase Average Trend method (PAT) and the Month for Cyclical Dominance (MCD) method. The use of DHP helps increase the stability of the cyclical estimation (Gyomai & Guidetti, 2012). DHP applies HP twice to achieve a smoother de-trended cycle; the first step de-trends while the second step smoothes. The process aims to remove two factors; long-term trends and high-frequency noise. Firstly, the method takes a large value setting of λ to remove the long-term trend; this retains the cycle frequencies and the high-frequency components. Secondly, the process aims to remove high-frequency noise by using a smaller value of λ, meaning that the cut-off frequencies are much higher, preserving the trend as part of the filter results.

2.3 Christiano-Fitzgerald (CF)

The Christiano-Fitzgerald filter (CF) introduced by Christiano and Fitzgerald (1999) formulates the de-trending and smoothing problem in the frequency domain. This method is a band-pass filter for a random walk time series based on a similar principle to the Baxter and King (BK) filter of Baxter and King (1999). The BK's filtered series is a symmetric approximation with no phase shifts; therefore, the method requires series trimming for symmetry and phase correctness. Hence, depending on the trim factor, some data at the end of the time series cannot be estimated. In contrast, CF is an asymmetric filter that estimates all filtered data points by the whole time series. Gyomai and Guidetti (2012); and Nilsson and Gyomai (2011) stated that CF outperformed on a long series over BK, but the two converge in the long run to the optimal filter, while in real-time applications, CF outperforms the BK filter. For these reasons, only CF was included in this study. CF can be calculated as:

$$\mathrm{c_t} = \mathrm{B_0 y_t} + \mathrm{B_1 y_{t+1}} + \ldots + \mathrm{B_{T+1-t} y_{T-1}} + \tilde{\mathrm{B}}_{\mathrm{T-t}} \mathrm{y_T} + \mathrm{B_1 y_{t-1}} + \ldots + \mathrm{B_{t-2} y_2} + \tilde{\mathrm{B}}_{t-1} \mathrm{y_1} \quad (6)$$

$$\mathrm{B_j} = \frac{\sin(jb) - \sin(ja)}{\pi j}, \ \mathrm{j} \geq 1, \quad (7)$$

$$\mathrm{B_0} = \frac{b-a}{\pi} \quad (8)$$

$$a = \frac{2\pi}{p_u} \quad (9)$$

$$\mathrm{b} = \frac{2\pi}{p_l} \quad (10)$$

$$\tilde{\mathrm{B}}_k = -\frac{1}{2}\mathrm{B_0} - \sum_{j=1}^{k=1}\mathrm{B}_j \quad (11)$$

where p_u and p_l represent the cut-offs of cycle length; cycles longer than p_l and shorter than p_u are preserved in the cyclical term c_t.

2.4 One-Sided Hodrick-Prescott (OneHP)

Owing to the criticisms of end-of-sample bias and spurious predictability in HP (Hamilton, 2018), the One-sided Hodrick-Prescott (OneHP) of Hodrick and Prescott (1997), is frequently applied for de-trending as the real-time version of the regular Hodrick-Prescott filter (HD). Rather than using all sample data, OneHP applies only data until t to estimate trend and cyclical components, rather than beyond t, as with HP. Consequently, OneHP does not suffer from spurious predictability or end-point biases.

$$\hat{\tau}_{t|t,\lambda} = \arg\min_{\tau_t}\Big(\min_{\tau_1,\dots,\tau_{t-1}} \Big(\sum_{s=1}^{t}(y_s - \tau_s)^2 + \lambda\sum_{s=2}^{t-1}(\tau_{s+1} - 2\tau_s + \tau_{s-1})^2\Big)\Big) \tag{12}$$

3. DATA AND METHODOLOGY

This study aims to identify a robust cyclical estimation method (de-trending and smoothing) focusing on measuring the revision of size, signal, and turning points. The study includes five methods—four frequently-used cycle extraction methods (HP, DHP, CF, and OneHP) and one proposed method, namely DoneHP.

3.1 Data and Data Preparation

The study considered the choice of Thailand's reference series, including the Gross Domestic Product at Constant Prices (GDP), Coincident Economic Index (CEI), Manufacturing Production Index of Value Added (MPIVA), and Manufacturing Production Value Index (MPI). The original data source published GDP in quarterly data, while CEI, MPIVA, and MPI, were presented as monthly data. The analysis aims to investigate quarterly and monthly data; therefore, the study converted GDP into a monthly frequency by linearly interpolating following the work of Gyomai and Guidetti (2012) and Tsouma (2010). The monthly data of CEI, MPIVA, and MPI, were converted to quarterly data using an average method.

All data in the analysis must be free of seasonal components. Fortunately, a seasonally adjusted series of GDP, CEI, and MPIVA were available. However, the source did not offer a seasonally adjusted series for MPI. Hence, the study conducted a seasonal adjustment of monthly MPI by X-13 (MPI_SA).

The period span of data from the sources was different. All ended at the same period, but the first data were different. The first available data for GDPSA was 1993Q1, for CCISA it was at 2000M1, while the first data available for MPIVASA and MPI were at 2011M1. Hence, the period span for the analysis was 2011Q1-2022Q1 and 2011M1 to 2022M3 for quarterly and monthly data, respectively. Table 1 shows the details of the data and data preparation for the study.

3.2 Cycle Extraction

The study included five methods of cycle extraction. Four of these were frequently-used cycle extraction methods, including HP, DHP, CF, and OneHP; the study also proposed the candidate method of DoneHP for this research.

DoneHP was conducted with the same concept as DHP. However, the method involves applying OneHP twice, in contrast to the regular HP since HP has some drawbacks as discussed by Hamilton (2018). λ was specified according to the following formula (OECD).

$$\lambda = \left[4\left(1-cos\left(\omega_0\right)\right)^2\right]^{-1} \tag{13}$$

$$\omega_0 = \frac{2\pi}{l} \tag{14}$$

where

λ is the smoothing parameter.

ω_0 is the frequency expressed in radians.

l denotes the number of periods it takes to complete a full cycle.

Following the methodology of OECD CLI, the study allows the removal of cyclical components with cycles longer than 120 months (40 quarters) and those with cycles

Table 1 Data and Data Preparation for the Research

	Data			
	Gross Domestic Product (Constant Prices)	Coincident Economic Index	Manufacturing Production Index (Value Added)	Manufacturing Production Value Index
Variable	GDP	CEI	MPIVA	MPI_SA
Period	1993Q1-2022Q1	2000M1-2022M3	2011M1-2022M3	2011M1-2022M3
Frequency	Quarterly	Monthly	Monthly	Monthly
Source	Office of the National Economic and Social Development Council	Bank of Thailand	Office of Industrial Economics	Office of Industrial Economics
Sample	2011Q1-2022Q1	2011M1-2022M3	2011M1-2022M3	2011M1-2022M3
SA Adjust Method	-	-	-	X-13
Frequency Converter	Chow-Lin for Monthly	Average for Quarterly	Average for Quarterly	Average for Quarterly

shorter than 12 months (4 quarters). Hence the λ values were 13.93 and 133,107.94 for the monthly data, and 0.25 and 1649.33 for quarterly data.

3.3 Standardization

After retaining the cyclical components, the results were standardized as the series were sometimes published in different units or scales. Using a standardized form ensures all units and scales will be the same, with a mean of 0 and a standard deviation of 1. All cyclical components were standardized by subtracting from the mean and dividing by the standard deviation.

3.4 Turning Point Detection

Turning points were identified by following the BryBoschan algorithm (Bry & Boschan, 1971), which is a simplified version of the National Bureau of Economic Research (NBER). A minimum period was set for the phase and cycle at five months (2 quarters) and 15 months (5 quarters), respectively; turning points outside these criteria would be eliminated. Additionally, peaks and troughs must alternate.

3.5 Expanding Windows

The expanding window experiment was applied to assess the stability of the cyclical data over time. The study checked for cyclical data revision to examine whether the cyclical data were time-invariant.

The expanding windows were a fixed starting point and incorporated the newly available data each time. Therefore, the size of the windows gradually increases the length of the time series.

As for the quarterly data, all the time series in the analysis had an opening window in 2011/Q1. However, the end of the windows was different. The end of the first window was in 2018Q4, and the last window ended in 2022/Q1. Regarding monthly data, 2011/M1 was the starting point for all windows, whereas 2018/M12 was the end of the first window, while the end of the last window was 2022/M3.

Cycle extraction was recursively re-estimated and recorded, moving forward in time using a set of observations from each expanding window. The number of windows (W) was dependent on the sample size (T), the first starting window size (m_1), and the size of the incremental expanding window.

Table 2 shows the results of the expanding window experiment. Data sample sizes of 45 and 135 were used for quarterly and monthly data, respectively. The first window of the quarterly data included 32 observations, while monthly data contained 45 observations. The incremental size between expanding windows, for both quarterly and monthly data was one period. Hence, 14 and 40 consecutive subsamples were obtained for quarterly and monthly data respectively. The cyclical components were recurrently estimated and recorded from the subsamples.

3.6 Evaluation

Differences in economic cycle results normally occur when new data are added to the study, since the cycle extraction is related to a time series analysis of the expanding window data using equations. The difference between the two results describes the same phenomenon and is called revision. In other words, revision refers to the historical values that change when the newly available data for subsequent periods is added to the analysis. The smaller relative revision reflects a more robust method. Therefore, the magnitude of the revision can be a criterion for evaluating the cycle extraction methods in terms of robustness.

Regarding the robust methods, the preferred cycle extraction method should give a smaller revision for each re-estimation. To pursue the research objectives, the relative performance of the cycle extraction methods was evaluated in terms of size, signal, and turning point.

The cycle extraction of the five methods was recursively examined with four variables using observations from each expanding window—5*4*14=280 estimation for quarterly data and 5*4*40=800 estimation for monthly data. For example, the data of expanding window i, starting from the observation of 1^{st} to $(m_1+i-1)^{th}$, was used to estimate the series of cyclical components i (C_t^i) (Figure 1).

3.6.1 The modification of each observation (t) was calculated from the difference between consecutive expanding windows. Revision of Size

This section aimed to measure the size of the revisions. Accordingly, the mean absolute revision and the standard deviation of revisions were calculated by:

$$R_t^i = C_t^{i+1} - C_t^i,\ i = 1,2,3,...,W-1 \qquad (15)$$

where

R_t^i is the revision of observation t at the i^{th} round.

C_t^i is the standardized cyclical component of observation t estimated from the expanding window i.

m_1 is the first window size.

W is the number of expanding windows.

Table 2: Results of the Expanding Window Experiment

	Quarterly	Monthly
Sample size (T)	45 (2011Q1-2022Q1)	135 (2011M1-2022M3)
First window size (m_1)	m_1=32 (2011Q1-2018Q4)	m_1=96 (2011M1-2018M12)
End window size ($m_i = m_1 + i - 1$)	m_{14}=45 (2011Q1-2022Q1)	m_{40}=135 (2011M1-2022M3)
Increments size	1	1
Number of Windows (W= T – m_1+ 1)	W=45-32+1=14	W=135-96+1=40

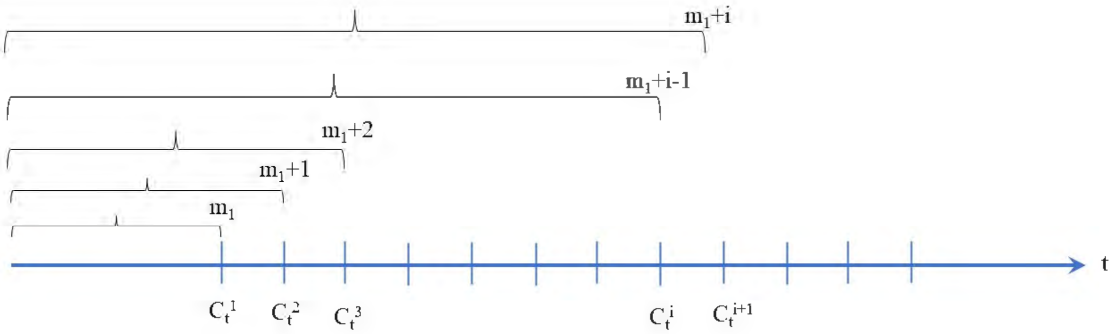

Figure 1: Diagramatic representation of the expanding window experiment

The mean of absolute revision measures the overall size of revisions regardless of whether the newly revised cyclical component was more or less than the previous version.

Mean absolute revision from round ith (MAR^i)

$$=\frac{\sum_{t=1}^{m_1+i-1}\left|R_t^i\right|}{m_1+i-1} \tag{16}$$

Average of mean absolute revision=

$$\overline{\text{MAR}}=\frac{\sum_{i=1}^{W-1} MAR^i}{W-1} \tag{17}$$

The standard deviation of revision measures the dispersal of the overall size of revisions from the revision mean. A low standard deviation infers the revision size is clustered around the mean, while a high standard deviation indicates the revision size is more spread out.

Standard deviation of revisions from round ith (SDR^i)

$$=\sqrt{\frac{\sum_{i=t}^{m_1+i-1}(R_t^i-\overline{R^i})^2}{m_1+i-2}} \tag{18}$$

Average of standard deviation of revisions =

$$\overline{SDR}=\frac{\sum_{i=1}^{W-1} SDR^i}{W-1} \tag{19}$$

3.6.2 Revision of Signal

This section focused on measuring a steady phase and direction signal. **The phase signal** is used to identify the cyclical phase, which is one of the key pieces of business cycle information. There are two important phases in a business cycle: prosperity and depression. The cyclical phase is recognized as the prosperity phase when the cyclical value is above the trend values; while, when the cyclical value is below the trend, it will be determined as the depression phase.

The number of phase signal changes was counted between the initial and the following estimation that was revised to shift from below trend to above trend or vice versa. There is no trend in this work since the methods aim to extract only the cyclical component; the cyclical value was also standardized so that the threshold was zero to measure the cyclical prosperity or depression phase as follows:

$$PhaseSignal_t^i=\begin{cases}1, C_t^i>0\\0, C_t^i=0\\-1, C_t^i<0\end{cases}, i=1,2,3,\ldots,W \tag{20}$$

$$R_t^i=\begin{cases}1, PhaseSignal_t^{i+1}\neq PhaseSignal_t^i\\0, PhaseSignal_t^{i+1}=PhaseSignal_t^i\end{cases}, \quad i=1,2,3,\ldots,W-1 \tag{21}$$

Phase Signal Change from round ith (PSC^i)

$$=\sum_{t=1}^{m_1+i-1} R_t^i \tag{22}$$

Average of Phase Signal Change = $\overline{PSC}$

$$=\frac{\sum_{i=1}^{W-1} PSC^i}{W-1} \tag{23}$$

A direction change is used to identify the direction of the cycle**.** When the cyclical value at t+1 increases from t, it is considered an expansion; while, when the cyclical value at t+1 decreases from t, it is viewed as a contraction. The revision of the direction change was measured by counting how many times the cyclical value changed to increase from decrease or vice versa between the initial and the following estimation.

$$DirectionSignal_t^i = \begin{cases} 1, C_{t+1}^i > C_t^i \\ 0, C_{t+1}^i = C_t^i \\ -1, C_{t+1}^i < C_t^i \end{cases}, \quad i = 1,2,3,\ldots,W \qquad (24)$$

$$R_t^i = \begin{cases} 1, DirectionSignal_t^{i+1} \neq DirectionSignal_t^i \\ 0, DirectionSignal_t^{i+1} = DirectionSignal_t^i \end{cases}, \quad i = 1,2,3,\ldots,W-1 \qquad (25)$$

Direction Signal Change from round ith (DSC^i)

$$= \sum_{t=2}^{m_1+i-1} R_t^i \qquad (26)$$

Average of Direction Signal Change =

$$\overline{DSC} = \frac{\sum_{i=1}^{W-1} DSC^i}{W-1} \qquad (27)$$

3.6.3 Revision of Turning Point

This section contains information on the measuring of the steadiness of dating turning points--peaks occur when the economy switches from recession to expansion, while troughs occur when the economy switches from expansion to recession. Identifying turning points is one of the ultimate goals in business cycle analysis, and is commonly used to determine the characteristics of an economic cycle, playing a role as a tool for policymakers to prevent economic downturns (Boldin, 1994). Due to the objective of evaluating the robustness of the cycle extraction method, in this section methods with less revision of dating turning points are preferred, and are identified by considering the mean and median of the absolute revision of the turning point.

Figure 2 provides an example to explain the calculation of turning point revision. The difference of turning points, both trough and peak, was calculated from the consecutive window i+1 and w. If there were extra turning points between those expanding windows, they would be ignored from the calculation. For instance, in Figure 2, there is one peak of window i+1 which is additional in respect of window i; hence, that peak was excluded from the analysis.

$$R\_Trough_{k_T}^i = Trough_{k_T}^{i+1} - Trough_{k_T}^i, \quad k_T = 1,2,\ldots,TP_T, \quad i = 1,2,3,\ldots,W-1 \qquad (28)$$

$$R\_Peak_{k_P}^i = Peak_{k_P}^{i+1} - Peak_{k_P}^i, \quad k_P = 1,2,\ldots,TP_P, \quad i = 1,2,3,\ldots,W-1 \qquad (29)$$

where

$R\_Trough_{k_T}^i$ is the revision of the trough k_T at the ith round.

$R\_Peak_{k_P}^i$ is the revision of the peak k_P at the ith round.

$Trough_{k_T}^i$ is the trough k_T estimated from expanding window i.

$Peak_{k_T}^i$ is the peak k_T estimated from expanding window i.

TP Mean of absolute revision from round ith (TP_MAR^i)

$$= \frac{\sum_{k=1}^{TP_T} \left| R\_Trough_{k_T}^i \right| + \sum_{k=1}^{TP_P} \left| R\_Peak_{k_P}^i \right|}{TP_T + TP_P} \qquad (30)$$

Average of TP Mean of absolute revision =

$$\overline{TP\_MAR} = \frac{\sum_{i=1}^{W-1} TP\_MAR^i}{W-1} \qquad (31)$$

4. EMPIRICAL RESULTS

The empirical results regarding the robustness of the five extraction methods, including HP, DHP, CF, OneHP, and DoneHP, are shown in this section. The five cycle extraction methods were evaluated for revision based on their relative performance in size, signal, and turning point, from the four variables: GDP, CEI, MPIVA, and MPI_SA. The cycle extraction method ranking was also

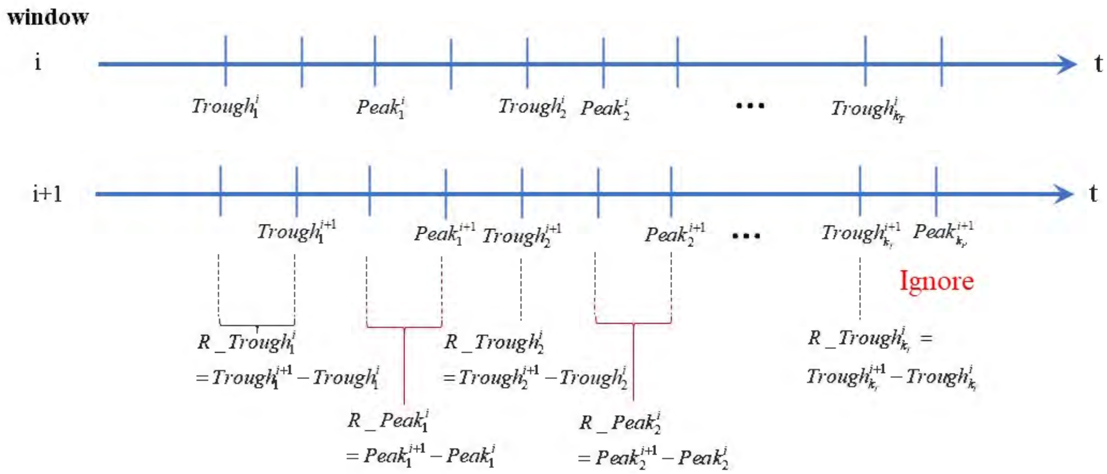

Figure 2: The Explanation of Calculating the Turning Point Revision

rated from 1-5 based on the method's performance (smallest to highest revision). The ranks (the bold font) in Tables 3, 4, and 5 were calculated from the rank average of the four variables.

Table 3 The Results of the Size Revision

		HP	DHP	CF	OneHP	DoneHP
Average of Mean Absolute Revision (Quarterly)	GDP	0.06 (3)	0.074 (4)	0.092 (5)	0.054 (2)	0.049 (1)
	CEI	0.032 (3)	0.04 (4)	0.062 (5)	0.027 (2)	0.018 (1)
	MPIVA	0.04 (2)	0.057 (4)	0.108 (5)	0.04 (3)	0.025 (1)
	MPI_SA	0.031 (3)	0.042 (4)	0.082 (5)	0.029 (2)	0.017 (1)
	Rank	**(3)**	**(4)**	**(5)**	**(2)**	**(1)**
Average of Mean Revision (Monthy)	GDP	0.019 (3)	0.027 (4)	0.038 (5)	0.015 (2)	0.015 (1)
	CEI	0.014 (3)	0.016 (4)	0.039 (5)	0.009 (2)	0.008 (1)
	MPIVA	0.015 (3)	0.023 (4)	0.057 (5)	0.012 (2)	0.01 (1)
	MPI_SA	0.012 (3)	0.017 (4)	0.049 (5)	0.009 (2)	0.007 (1)
	Rank	**(3)**	**(4)**	**(5)**	**(2)**	**(1)**
Table 3 (continued)						
Standard Deviation of Revision (Quarterly)	GDP	0.093 (3)	0.113 (4)	0.118 (5)	0.055 (2)	0.042 (1)
	CEI	0.049 (3)	0.064 (4)	0.087 (5)	0.019 (2)	0.008 (1)
	MPIVA	0.064 (3)	0.098 (4)	0.149 (5)	0.031 (2)	0.015 (1)
	MPI_SA	0.049 (3)	0.071 (4)	0.113 (5)	0.02 (2)	0.01 (1)
	Rank	**(3)**	**(4)**	**(5)**	**(2)**	**(1)**
Standard Deviation of Revision (Monthy)	GDP	0.037 (3)	0.048 (4)	0.05 (5)	0.016 (2)	0.013 (1)
	CEI	0.027 (3)	0.032 (4)	0.053 (5)	0.004 (2)	0.004 (1)
	MPIVA	0.031 (3)	0.049 (4)	0.077 (5)	0.007 (2)	0.006 (1)
	MPI_SA	0.025 (3)	0.038 (4)	0.067 (5)	0.005 (2)	0.004 (1)
	Rank	**(3)**	**(4)**	**(5)**	**(2)**	**(1)**

Note: () represents the ranking of methods from less (1) to high revision (5)

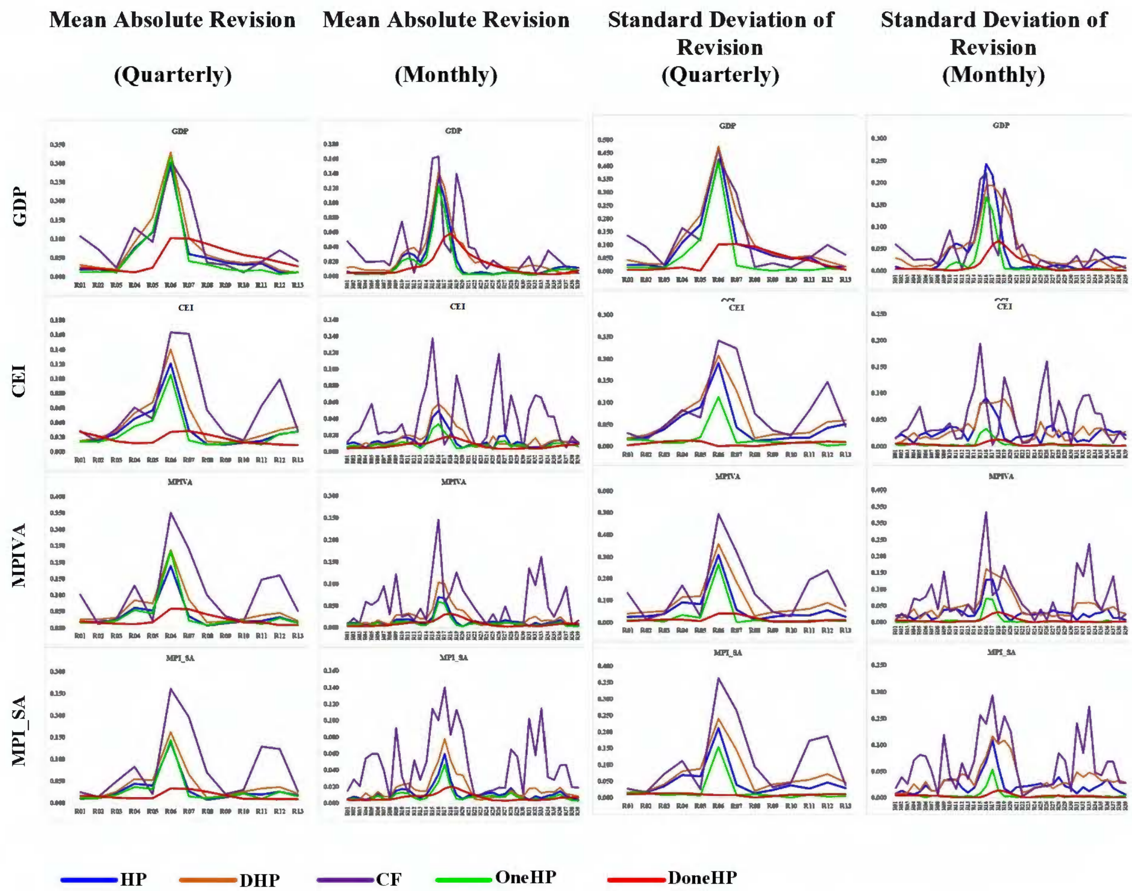

Figure 3: The Results of the Size Revision

The results led to the consensus that DoneHP was outstanding from other methods. The procedures' performance order was DoneHP, OneHP, DHP, HP, and CF, based on the overall ranking.

Moreover, this analysis showed that a large revision happened during 2019Q4 to 2020Q4 (R5-R8) and 2019M12 to 2020M12 (R25-R39) for quarterly and monthly data. This big revision of cycle extraction occurred during the early stage of COVID-19 in 2020.

4.1 The Results of Size Revision

Regarding the measure of size revision (the mean absolute and the standard deviation of the revision), the cycles estimated by DoneHP outperformed other methods. This procedure gained the least modification, followed by OneHP, HP, DHP, and CF. Table 3 shows the average of the mean absolute revision and standard deviation of the revision from all windows. Figure 3 expresses the mean absolute revision, including the standard deviation of the revision from each round i (R_i).

4.2 The Results of Signal Revision

Regarding the signal consideration concerning phase and direction of change, it was also DoneHP that held the first ranking and the smallest revisions, followed by OneHP, DHP, HP, and CF. The average of the phase signal change and direction of signal change from all windows is shown in Table 4. Figure 4 shows the revision results regarding each round's phase and direction change between consecutive windows.

Table 4: The Results of the Signal Revision

		HP	DHP	CF	OneHP	DoneHP
Phase Signal (Quarterly)	GDP	0.462 (1)	0.462 (1)	0.538 (3)	0.692 (5)	0.538 (3)
	CEI	0.385 (3)	0.538 (4)	0.923 (5)	0.231 (2)	0.077 (1)
	MPIVA	0.462 (2)	0.538 (3)	1.077 (5)	0.692 (4)	0.154 (1)
	MPI_SA	0.846 (5)	0.462 (1)	0.769 (4)	0.538 (3)	0.462 (1)
	Rank	**(3)**	**(2)**	**(5)**	**(4)**	**(1)**
Phase Signal (Monthly)	GDP	0.821 (3)	0.564 (1)	0.974 (5)	0.821 (3)	0.667 (2)
	CEI	1.000 (4)	0.923 (3)	1.231 (5)	0.436 (2)	0.282 (1)
	MPIVA	0.769 (3)	0.795 (4)	1.744 (5)	0.718 (2)	0.359 (1)
	MPI_SA	0.718 (3)	0.769 (4)	1.769 (5)	0.667 (2)	0.436 (1)
	Rank	**(4)**	**(3)**	**(5)**	**(2)**	**(1)**
Direction Signal (Quarterly)	GDP	0.231 (3)	0.308 (4)	1.077 (5)	0.000 (1)	0.000 (1)
	CEI	0.154 (3)	0.231 (4)	0.692 (5)	0.000 (1)	0.000 (1)
	MPIVA	0.231 (4)	0.154 (3)	0.846 (5)	0.000 (1)	0.000 (1)
	MPI_SA	0.385 (4)	0.077 (3)	0.538 (5)	0.000 (1)	0.000 (1)
	Rank	**(3)**	**(3)**	**(5)**	**(1)**	**(1)**
Direction Signal (Monthly)	GDP	0.487 (3)	0.795 (4)	1.590 (5)	0.000 (1)	0.000 (1)
	CEI	0.077 (3)	0.359 (4)	1.077 (5)	0.000 (1)	0.000 (1)
	MPIVA	0.077 (3)	0.282 (4)	1.615 (5)	0.000 (1)	0.000 (1)
	MPI_SA	0.051 (3)	0.436 (4)	1.821 (5)	0.000 (1)	0.000 (1)
	Rank	**(3)**	**(4)**	**(5)**	**(1)**	**(1)**

Note: () represents the ranking of methods from less (1) to high revision (5)

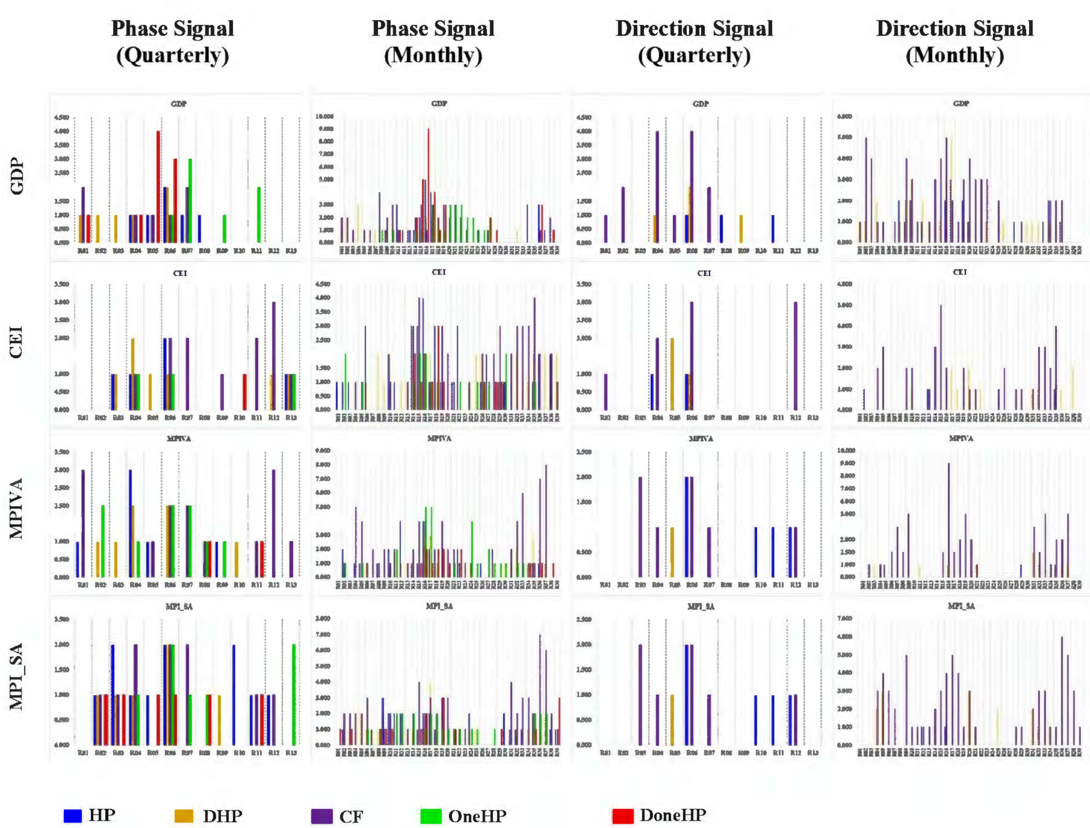

Figure 4: The Results of Signal Revision

4.3 The Result of Turning Point Revision

Finally, the measuring of the turning point revision indicated that both DoneHP and OneHP were in the first rank with smaller modifications, followed by DHP, HP, and CF.

Table 5 shows the average from all windows of the mean and median absolute revision of the turning points; Figure 5 displays the mean and median absolute revision of the turning point from each round i.

4.4 The Overall Result of the Revisions

Table 6 summarizes all size, signal, and turning point, revision results by calculating their total summation and consequent ranking. The results clearly show that the performance of DoneHP is outstanding, followed by OneHP, DHP, HP, and CF (Table 6).

Table 5: The Results of the Turning Point Revision

		HP	DHP	CF	OneHP	DoneHP
Mean Absolute Revision of Turning Point (Quarterly)	GDP	2.000 (5)	0.077 (3)	0.474 (4)	0.000 (1)	0.000 (1)
	CEI	0.000 (1)	0.173 (5)	0.101 (4)	0.000 (1)	0.000 (1)
	MPIVA	0.123 (4)	0.112 (3)	0.799 (5)	0.000 (1)	0.000 (1)
	MPI_SA	0.054 (4)	0.013 (3)	0.058 (5)	0.000 (1)	0.000 (1)
	Rank	**(3)**	**(3)**	**(5)**	**(1)**	**(1)**
Mean Absolute Revision of Turning Point (Monthly)	GDP	0.072 (3)	0.391 (4)	1.158 (5)	0.000 (1)	0.000 (1)
	CEI	0.106 (3)	0.188 (5)	0.134 (4)	0.000 (1)	0.000 (1)
	MPIVA	1.624 (5)	0.064 (3)	0.881 (4)	0.000 (1)	0.000 (1)
	MPI_SA	0.129 (3)	0.314 (5)	0.169 (4)	0.000 (1)	0.000 (1)
	Rank	**(3)**	**(4)**	**(4)**	**(1)**	**(1)**
Median Absolute Revision of Turning Point (Quarterly)	GDP	2.000 (5)	0.000 (1)	0.231 (4)	0.000 (1)	0.000 (1)
	CEI	0.000 (1)	0.000 (1)	0.038 (5)	0.000 (1)	0.000 (1)
	MPIVA	0.000 (1)	0.000 (1)	0.846 (5)	0.000 (1)	0.000 (1)
	MPI_SA	0.000 (1)	0.000 (1)	0 (1)	0.000 (1)	0.000 (1)
	Rank	**(4)**	**(1)**	**(5)**	**(1)**	**(1)**
Median Absolute Revision of Turning Point (Quarterly)	GDP	0.000 (1)	0.000 (1)	0.603 (5)	0.000 (1)	0.000 (1)
	CEI	0.000 (1)	0.000 (1)	0.026 (5)	0.000 (1)	0.000 (1)
	MPIVA	1.808 (5)	0.000 (1)	0.897 (4)	0.000 (1)	0.000 (1)
	MPI_SA	0.000 (1)	0.000 (1)	0 (1)	0.000 (1)	0.000 (1)
	Rank	**(4)**	**(1)**	**(5)**	**(1)**	**(1)**

Note: () represents the ranking of methods from less (1) to high revision (5)

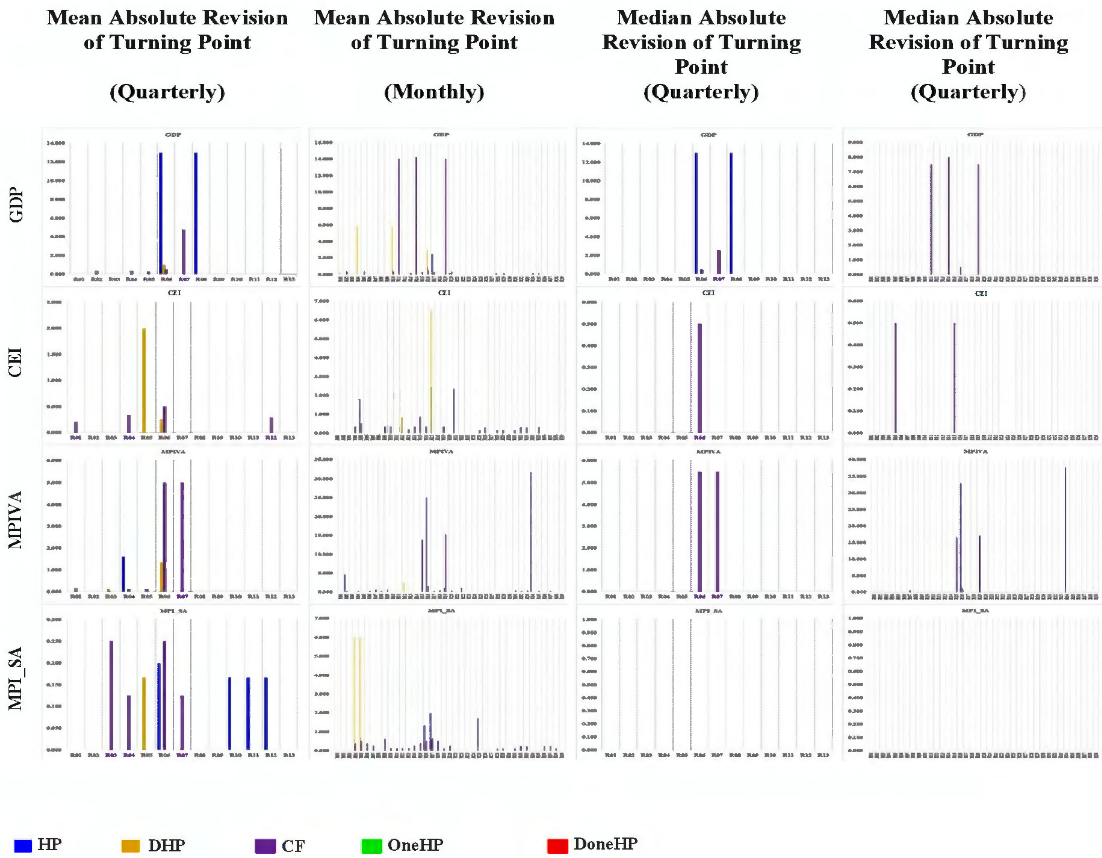

Figure 5: The results of the turning point revision

Table 6 The Rank Summary by Revisions

		HP	DHP	CF	ONEHP	ONEDHP
Rank summation	Size (Quarterly)	(6)	(8)	(10)	(4)	(2)
	Size (Monthly)	(6)	(8)	(10)	(4)	(2)
	Total Rank	*(12)*	*(16)*	*(20)*	*(8)*	*(4)*
	Size Rank	**(3)**	**(4)**	**(5)**	**(2)**	**(1)**
Rank summation	Signal (Quarterly)	(6)	(5)	(10)	(5)	(2)
	Signal (Monthly)	(7)	(7)	(10)	(3)	(2)
	Total Rank	*(13)*	*(12)*	*(20)*	*(8)*	*(4)*
	Signal Rank	**(4)**	**(3)**	**(5)**	**(2)**	**(1)**
Rank summation	Turning Point (Quarterly)	(7)	(4)	(10)	(2)	(2)
	Turning Point (Monthly)	(7)	(5)	(9)	(2)	(2)
	Total Rank	*(14)*	*(9)*	*(19)*	*(4)*	*(4)*
	Turning Point Rank	**(4)**	**(3)**	**(5)**	**(1)**	**(1)**
	Total Rank of Size, Signal, and Turning Point	*(39)*	*(37)*	*(59)*	*(20)*	*(12)*
Overall Rank		(4)	(3)	(5)	(2)	(1)

Note: () represents the ranking of methods from less (1) to high revision (5)

4.5 The Cycle Comparison of the Estimation from the First and the Last Windows.

All results above show the revisions between consecutive windows of the same cyclical value estimate. For this section, the study displays the cumulative revision of the cyclical value. Figures 6 and 7 compare the first estimate from the initial window and the final estimate from the last window. Roughly, the result shows that at the end of the cycle, HP and DHP are highly different, supporting the criticism of Hamilton (2018). Meanwhile, DoneHP and OneHP gained minimal revision of cycle values, especially at the end of the cycle.

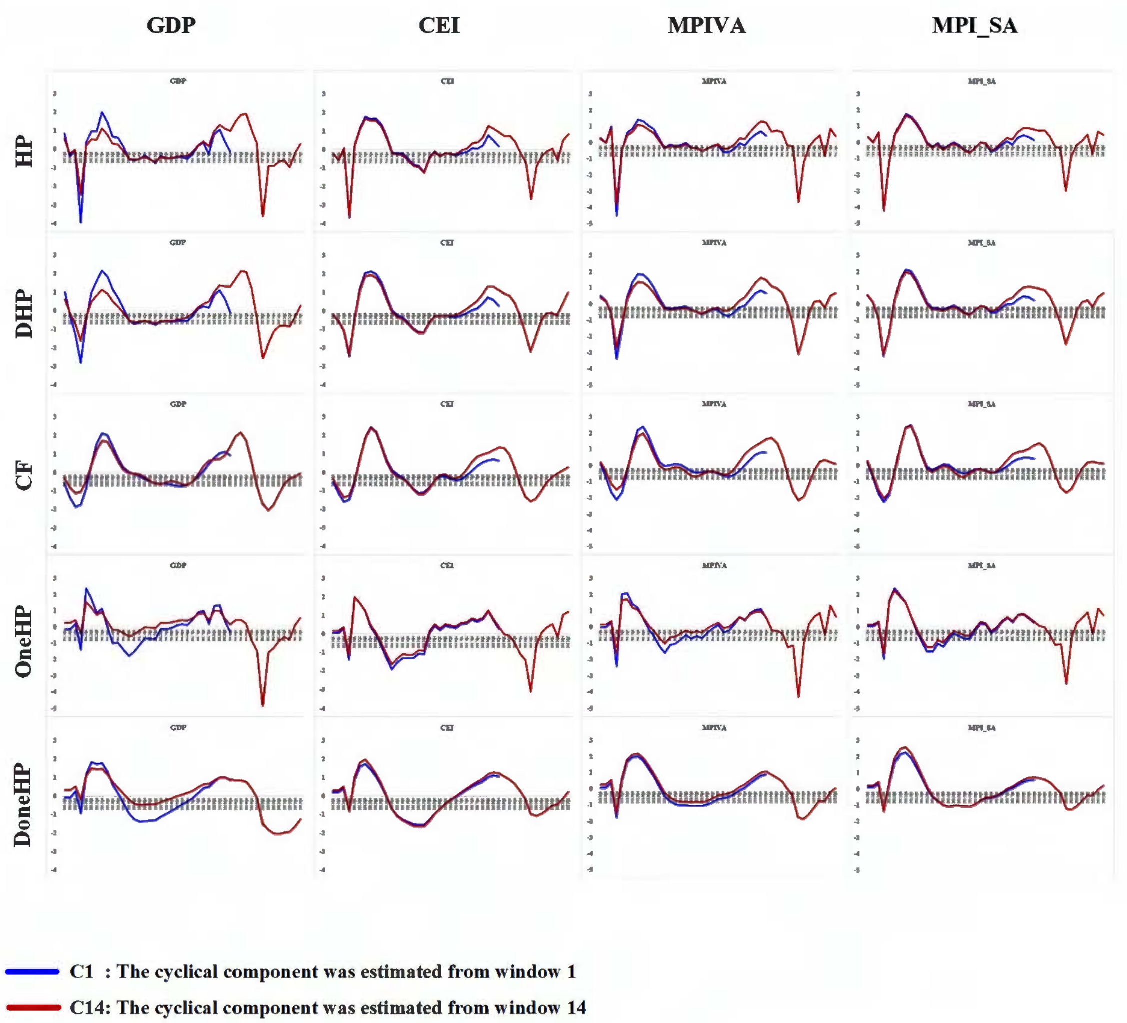

Figure 6: The comparison of the cycle estimated from the first and the last windows of the quarterly data.

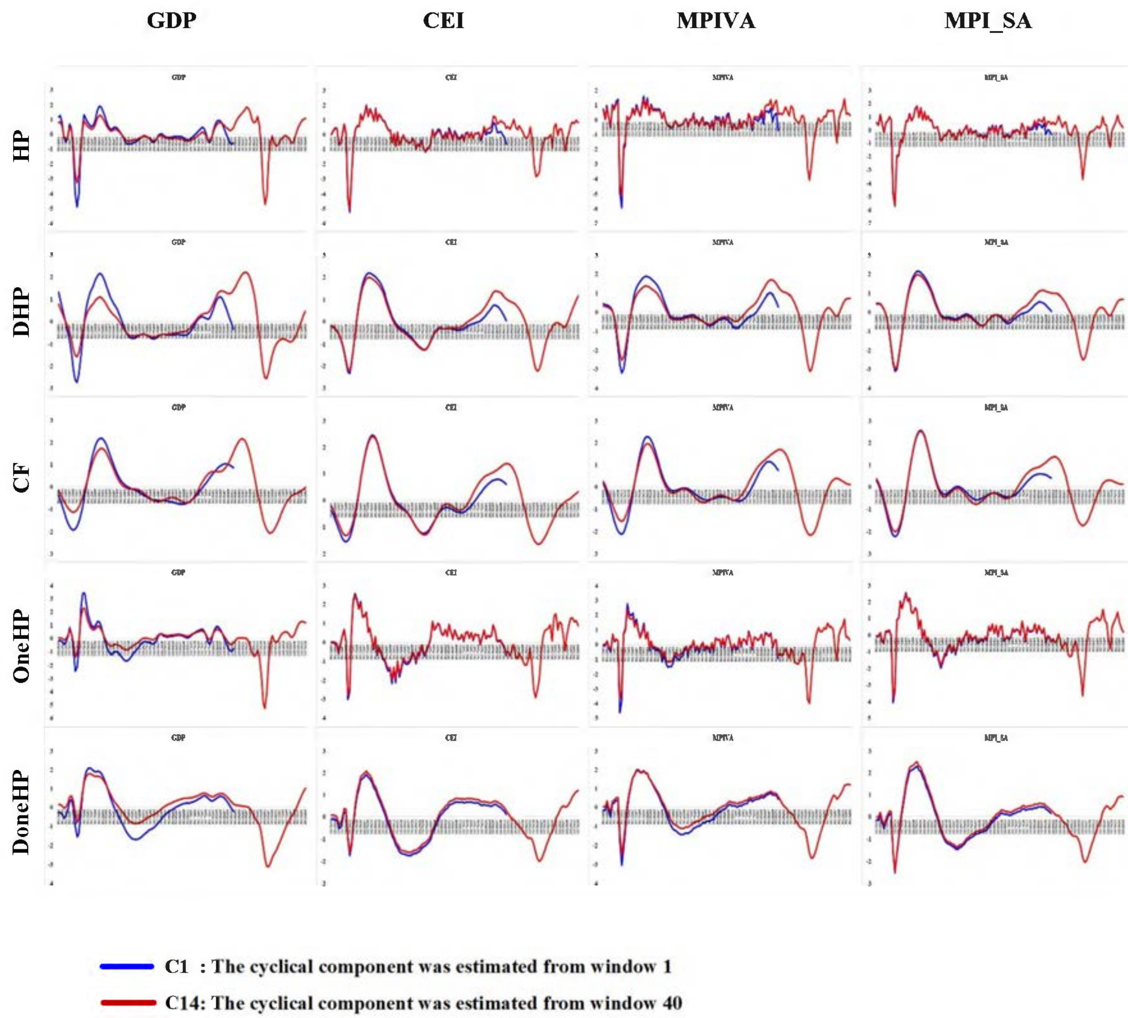

Figure 7: The comparison of the cycle estimated from the first and the last windows of the monthly data

5. CONCLUSION AND RECOMMENDATIONS

The research aimed to identify the robust cycle extraction method concerning the size, signal, and turning points, of the revision properties. Five cycle extraction methods were involved in the study, including the Hodrick-Prescott (HP), Double Hodrick-Prescott (DHP), Christiano-Fitzgerald (CF), One-sided Hodrick-Prescott (OneHP), and Double One-sided Hodrick-Prescott (DoneHP). The first four methods are frequently used for cycle extraction, while the fifth is the proposed method of this research.

Eventhough HP is one of the most commonly used; the OECD CLI system also applied HP twice for cycle extraction, called DHP (Gyomai & Guidetti, 2012; OECD, 2023). However, criticism of using HP has been spread regarding the results it generates, such as spurious dynamic relations (spurious predictability) and end-point problems (Hamilton, 2018). Hence, this study proposed applying DoneHP using OneHP twice instead of HP.

These methods were rated on their revision performance in an expanding window experiment. A less relative revision reflects a more robust technique. The consensus of the results of the revision according to size, signal, and turning points, indicates that the performance of DoneHP is outstanding from the others. Based on the overall ranking, the order of performance for the methods was DoneHP, OneHP, DHP, HP, and CF. In addition, the rough analysis of the cumulative revision also shows that DoneHP and OneHP gained minimal modification of cycle values, especially at the end of the cycle.

Hence, the research supports applying DoneHP for cycle extraction if the analysis is made in real-time and prefers not to revise the past estimates. In detail, the research suggests DoneHP followed by OneHP, HP, DHP, and CF if the priority is minimizing size revisions. However, when emphasis focuses on a steady signal and turning points; the study also suggests DoneHP, followed by OneHP, DHP, HP, and CF.

DoneHP is therefore recommended for cycle extraction when studying the economic cycle, especially in Thailand, paving the way to predict the economic cycle by exploring the leading indicator and developing an economic early warning model. DoneHP is also recommended for estimating the long-term trend in key macroeconomic factors, i.e., GDP, CEI, and MPI, to compute the long-term trend. For example, calculating the potential long-term GDP applied to estimate the output gap, a key variable for a monetary policy decision.

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