

ESTIMATING DHAKA STOCK MARKET VOLATILITY: A COMPARISON BETWEEN STANDARD AND ASYMMETRIC GARCH MODELS

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Abstract

This paper compares and estimates standard and asymmetric GARCH models with daily returns data of the DSI index (All Share Price Index) of the Dhaka Stock Exchange from 28 March 2005 to 30 November 2010. The maximum likelihood estimation (MLE) technique is used to estimate the parameters of the chosen models. Results indicate that GARCH has lower log-likelihood than the asymmetric GJR-GARCH model, which implies that GJR-GARCH model is a better performing model to estimate and to forecast volatility. Results from other hypothesis tests indicate that volatility process has unit root i.e. volatility process is nonstationary, expected returns do not always depend on volatility, and the conditional variance (volatility) of future asset price is a symmetric function of changes in price at Dhaka stock market.

บทคัดย่อ

งานวิจัยนี้เปรียบเทียบและประเมินแบบจำลอง GARCH แบบมาตรฐานและแบบอสมมาตร โดยใช้ข้อมูลผลตอบแทนรายวันของดัชนี DSI จากตลาดหลักทรัพย์ธากา (Dhaka Stock Exchange) ประเทศบังกลาเทศตั้งแต่วันที่ 28 มีนาคม 2548 ถึง 30 พฤศจิกายน 2553 งานวิจัยนี้ใช้เทคนิคตัวประมาณความน่าจะเป็นสูงสุด (MLE) ในการประมาณค่าพารามิเตอร์ของแบบจำลอง ผลลัพธ์แสดงให้เห็นว่าแบบจำลอง GARCH แบบมาตรฐานมีความน่าจะเป็น (log-likelihood) ที่ต่ำกว่าแบบจำลอง GJR-GARCH แบบอสมมาตรซึ่งหมายความว่ารูปแบบ GJR-GARCH จะมีประสิทธิภาพดีกว่ารูปแบบ GARCH ในการประเมินและคาดการณ์ความผันผวนของผลตอบแทน ผลลัพธ์ที่ได้จากการทดสอบสมมติฐานอื่นๆ ระบุว่า ความผันผวนของผลตอบแทนมี Unit Root หรือกล่าวได้ว่าความผันผวนมีลักษณะไม่นิ่ง และผลตอบแทนที่คาดว่าจะได้รับไม่ได้ขึ้นอยู่กับความผันผวนและความแปรปรวนของราคาเสมอไป นอกจากนี้ยังพบว่าความแปรปรวนหรือความผันผวนแบบมีเงื่อนไขของราคาสินทรัพย์ในอนาคตเป็นฟังก์ชันสมมาตรของการเปลี่ยนแปลงในราคาหุ้นของตลาดหลักทรัพย์ธากา

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I. INTRODUCTION

Modelling and forecasting volatility in financial markets has received much attention among researchers in the last twenty years. Researchers conduct both theoretical and empirical investigations of volatility forecasting. The key reason behind the extensive research of the volatility is that it plays a prominent role in financial markets. Volatility is associated with the key attributes in investing, security valuation, risk and uncertainty, risk management, derivative securities pricing, and monetary policy making. Taylor (2005) suggests that volatility can be defined and interpreted in different ways. One of the ways is volatility can be considered as a conditional volatility which is the standard deviation of a future return that is conditional on known information such as the history of previous returns. A time-series model, selecting and estimating using appropriate data, can calculate the expectation for the next period. The class of ARCH models provides convenient and accurate equations for volatility expectations. These ARCH class models are autoregressive, conditional heteroskedastic models that specify the conditional variance of the return in a specific period. One of the ARCH class models is generalized autoregressive conditional heteroskedasticity, GARCH (1, 1), which is a weighted sum of squared excess returns called innovations. In this paper, we choose two classes of ARCH models to estimate volatility. We compare between the standard generalized autoregressive conditional heteroskedasticity (GARCH) and Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) models.

The class of ARCH models for forecasting volatility has been used in financial econo-

metrics literature. Empirical findings suggest that GARCH is the most popular structure in financial time series. However, some studies infer that GARCH models are still poor in forecasting volatility compared to historical and implied volatility models while some other studies do not find clear-cut results.

An earlier investigation to test the predictive power of GARCH appears in Akgiray (1989). He compares the forecasts of monthly variance obtained from four forecasting methods: Historical estimate, Exponentially Weighted Moving Average (EWMA) forecast, ARCH forecast, and GARCH forecast. He finds that GARCH consistently outperforms all forecasting methods and GARCH forecasts are less biased and more accurate. Pagan and Schwert (1990) compare between GARCH and Exponential GARCH (EGARCH) models. Lee (1991) and Randolph and Najand (1991) compare GARCH with other volatility models. Both papers suggest that GARCH does not work as well because it tends to provide persistent forecast, which is valid only in period when changes in volatility are small. Brailsford and Faff (1996) examine the ability of GARCH and GJR-GARCH models along with other volatility forecasting models to forecast the aggregate monthly stock market volatility in Australia. Their results suggest that GJR-GARCH specification is the superior fitting model in forecasting volatility. However, Franses and Dijk (1996), Brooks (1998), Bali (2000), and Pong et al. (2004) do not recommend GJR-GARCH model in forecasting volatility. Taylor et al. (2010) finds that although model-free volatility expectation and at-the-money (ATM) implied volatility outperform other volatility models, asymmetric GARCH model is better for short forecast

horizons.

The rest of the paper is organized as follows. Section II reviews the standard GARCH and GJR-GARCH models. Datasets are discussed in section III. Various results such as parameter estimates from different volatility models and hypothesis tests are reported in section IV. Section V concludes.

II. MODELS

Generalized Autoregressive Conditional Heteroskedasticity (GARCH)

The most popular ARCH specification is the generalized autoregressive conditional heteroskedasticity (GARCH) suggested by Bollerslev (1986) and Taylor (1986). GARCH model includes a lagged variance term in the conditional variance equation which is defined by:

$$h_t = \omega + \alpha(r_{t-1} - \mu)^2 + \beta h_{t-1} \quad (1)$$

This model has four parameters, μ , α , β and ω . These parameters are constrained by $\omega \geq 0$, $\alpha \geq 0$, and $\beta \geq 0$. It assumes that conditional variance is never negative. The major properties of a GARCH stochastic process reported by Taylor (2005) are:

- i. The process is stationary if $\alpha + \beta < 1$
- ii. The unconditional variance is finite
- iii. The unconditional kurtosis always exceeds three and can be infinite
- iv. There is no autocorrelation between r_t and $r_{t+\tau}$ for all $\tau > 0$
- v. The correlation between the squared residual $s_t = (r_t - \mu)^2$ and $S_{t-\tau}$ is positive for all $\tau > 0$ and equals $C(\alpha + \beta)^\tau$, with C positive and determined by both α and β .

The excess return innovations are:

$$\varepsilon_t = r_t - \mu$$

and the standardized residuals are:

$$z_t = \frac{r_t - \mu}{\sqrt{h_t}} \quad (2)$$

$z_t \sim i.i.d N(0,1)$

The GARCH model is popular in empirical research because of its smaller number of parameters (only four) and it can be estimated easily. However, the apparent success of these simple parameterizations of the GARCH model cannot capture some important features of data such as leverage or asymmetric effect. Asymmetric effect implies that a fall in the stock market has a different impact on the next day's volatility than a rise of the same magnitude. Nelson (1991) identifies at least three major drawbacks of this model: (i) there may be negative correlation between current and future returns volatility but GARCH models rule this out by assumption, (ii) parameter restrictions in GARCH models are often violated by estimated coefficients and may unduly restrict the dynamics of the conditional variance process, and (iii) it is difficult to interpret whether shocks to conditional variance persist in GARCH models because the usual norms measuring persistence often do not agree. Glosten, Jagannathan and Runkle (1993) develop GJR-GARCH models that capture the asymmetric effects. The GJR-GARCH model is discussed next.

Glosten-Jagannathan-Runkle GARCH (GJR-GARCH)

Asymmetry in the GJR-GARCH can be introduced by weighting ε_{t-1}^2 differently for negative and positive residuals:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \alpha^- S_{t-1} \varepsilon_{t-1}^2 + \beta h_{t-1} \quad (3)$$

where S_{t-1} is the dummy variable which equals to

$$\begin{cases} 1 & \text{if } \varepsilon_{t-1} < 0 \\ 0 & \text{if } \varepsilon_{t-1} \geq 0 \end{cases}$$

Returns (ignoring dividends) r_t and conditional means μ_t are defined by:

$$r_t = \log(p_t / p_{t-1}) = \mu_t + \varepsilon_t = \mu_t + h_t^{1/2} z_t \quad (4)$$

$$\mu_t = \mu + \lambda \sqrt{h_t} + \theta \varepsilon_{t-1} \quad (5)$$

III. DATA

Daily returns data is used to model the volatility. The DSI index (All Share Price Index) of the Dhaka Stock Exchange is chosen to test the volatility models. Data are available for approximately six years from 28 March 2005 to 30 November 2010 inclusive when holidays and weekends are excluded. There are 1,368 observations for the index. The daily returns are calculated as:

$$r_t = \ln(p_t / p_{t-1}) \quad (6)$$

where p_t and p_{t-1} are current price and lag-price respectively.

IV. ESTIMATION RESULTS AND HYPOTHESIS TESTS

This section reports and discusses the results of the parameter estimation of the DSI index with daily return series to compare and demonstrate the empirical properties of the two GARCH models. One of the GARCH models is the asymmetric volatility model. The datasets cover the full sample period from 28 March 2005 to 30 November 2010.

Since the main focus of this paper is con-

ditional variance, rather than conditional mean, we concentrate on the unpredictable part of the stock returns. Using the unpredictable stock return series as the data series, we estimate the standard GARCH model as well as another parametric model, GJR-GARCH (Glosten-Jagannathan-Runkle) model, which is capable of capturing the leverage or size effects. These two models are reported for the conditional normal distributions. The estimation is performed using the maximum likelihood approach.

Table 1: Estimation Results of the DSI Index (All Share Price Index), Daily Returns

This table reports the estimation results of two predictable volatility models for the daily return of the DSI index (All Share Price Index) of the Dhaka Stock Exchange. The estimation is performed by the method of maximum likelihood. The sample period is from 28 March 2005 to 30 November 2010, when weekend and holidays are excluded. In the estimation results part of the table, the numbers in parentheses (.) are the standard errors.

h_t is the conditional variance on period t and ε_{t-1} is the unpredictable return on period $t-1$. Parameter estimations are done for the following models:

GARCH:

$$h_t = \omega + \alpha(r_{t-1} - \mu)^2 + \beta h_{t-1}$$

GJR-GARCH:

$$h_t = \omega + \alpha \varepsilon_{t-1}^2 + \alpha^- S_{t-1} \varepsilon_{t-1}^2 + \beta h_{t-1}$$

where

$$\varepsilon_{t-1} = r_{t-1} - \mu_{t-1}, r_t = \log(p_t / p_{t-1}) = \mu_t + \varepsilon_t = \mu_t + h_t^{1/2} z_t, \text{ and } \mu_t = \mu + \lambda \sqrt{h_t} + \theta \varepsilon_{t-1}$$

Parameter	GARCH	GJR-GARCH
$\mu \times 10^3$	1.1222	1.1352
$\lambda \times 10^2$		-0.0086 (1.740623)
θ		0.1147 (0.029366)
$\omega \times 10^7$	0.2584 (0.0000)	0.2557 (0.0000)
α	0.0001 (0.000008)	0.0001 (0.000213)
α^-		0.0000 (0.000949)
β	0.9997 (0.000056)	0.9997 (0.000106)
ϕ	0.9998 (0.000056)	0.9998 (0.000106)
$\sigma^2 \times 10^3$	2.5844	2.5128
Log L	4011.56	4015.71

Notes: Various constraints are used to estimate the parameters of the selected models. Those are--- $\omega \geq 0$, $\alpha \geq 0.0001$, $\alpha + \alpha^- \geq 0$, $\beta \geq 0$, $\phi \leq 0.9999$, $\sigma^2 \geq 0$. Log-likeli-

hood, $\log L(\theta) = \sum_{t=1}^n l_t(\theta)$ where

$$l_t(\theta) = -\frac{1}{2} \log(2\pi) - \frac{1}{2} \log(h_t(\theta)) - \frac{z_t^2}{h_t(\theta)}$$

for GARCH and GJR-GARCH-normal.

Table 1 presents the parameter estimates of the selected models, standard errors, and log-likelihoods for the daily DSI index. The estimation results in Table 1 indicate that all

the parameters, except λ , are significant using the standard errors. λ is negative and insignificant estimate. This implies that the relationship between the conditional mean and conditional variance h_t in Dhaka stock market is not reasonable. The parameter corresponding to the $S_{t-1} \varepsilon_{t-1}^2$ term in the GJR-GARCH model is significant but approximately zero using the standard errors. It is interesting to compare the estimate of α with the estimates of α in the GJR-GARCH model. With $\alpha = 0.0001$ and $\alpha^- = 0$, the squares of positive and negative residuals are multiplied by $\alpha = 0.0001$ and $\alpha + \alpha^- = 0.0001$ respectively. This indicates that positive and negative residuals have the same impact on the conditional variance in the DSI index returns. This result is inconsistent with most stock market studies where negative residuals have more impact on the conditional variance than do positive residuals.

It can be seen that GARCH has lower log-likelihood than the asymmetric GJR-GARCH model. This indicates that the GJR-GARCH model outperforms the standard GARCH model in capturing the dynamic behaviour of the DSI index returns.

To further understanding of these two different volatility models, some summary statistics, including the mean, standard deviation, minimum, maximum, skewness and kurtosis, are produced for each of the conditional variance series and are reported in Table 2.

Table 2: Summary Statistics of the Conditional Variance Estimate

This table reports the summary statistics of the estimated conditional variances of the DSI index (All Share Price Index) of the Dhaka Stock Exchange. Daily returns data are used. The sample period is from 28 March 2005 to 30 November 2010, when weekend and holidays are excluded.

	Mean $\times 10^3$	Standard Deviation $\times 10^5$	Skewnes	Kurtosis	Minimum $\times 10^3$	Maximum $\times 10^3$
GARCH	0.1558	0.5181	0.4472	-1.0016	0.1479	0.1664
GJR-GARCH	0.1703	0.3138	0.6498	-0.7895	0.1654	0.1760

Test for the Unit Root in Volatility

We test the null hypothesis $H_0: \phi = 1$ against the alternative of interest $H_1: \phi < 1$

The following test results are for the volatility persistence using the GJR-GARCH model. The log likelihood $\text{Log } L(\theta)$ is 4015.71. When the constraint $\phi = 1$ is applied, $\text{Log } L(\theta_0) = 4015.17$. The likelihood ratio (LR) test can be defined by the following equation:

$$\zeta_{LR} = -2(\log L(\theta_0) - \log L(\theta)) \quad (7)$$

where $\log L(\theta_0)$ and $\log L(\theta)$ indicate the log likelihoods for null and alternative hypotheses. The test statistic is compared with a critical value from the χ^2 distribution. Thus the likelihood ratio (LR) test value is $-2(4015.17 - 4015.71) = 1.08$. Comparing with the χ^2_1 distribution, the null cannot be rejected i.e. there is evidence in favour of unit root in the Dhaka stock market volatility. This implies that returns have permanent impact on future volatility.

Testing the Relationship Between Expected Returns and Volatility

A positive relationship is apparently reasonable between the conditional mean μ_t and conditional variance h_t from the following equation:

$$\mu_t = \mu - \lambda \sqrt{h_t} + \theta e_{t-1}$$

We test the null hypothesis $H_0: \lambda = 0$

Against the alternative of interest $H_1: \lambda > 0$

The log-likelihood $\text{Log } L(\theta)$ is 4015.71. When the constraint $\lambda = 0$ is applied the $\text{Log } L(\theta_0) = 4015.70$. Hence the LR test value is $-2(4015.70 - 4015.71) = 0.02$. This value is compared with the χ^2_1 distribution. Performing the Wald test, the absolute value of the test statistic is $\frac{\hat{\lambda}}{se(\hat{\lambda})}$ where $se(\lambda)$ is the stan-

dard error of λ . The Wald test statistic is compared with the χ^2 distribution with v degrees of freedom. The Wald test statistic is the $-4.92892\text{E-}05$ which is compared with a critical value from the normal distribution. Both the LR and the Wald tests indicate that null hypothesis cannot be rejected at any common levels of significance which implies that λ is not different from zero. Since here λ is statistically zero, we can say that it is insignificant. However, there is a controversy about the sign and significance of λ . We find in our study on Dhaka stock market that λ is negative and insignificant estimate. Our finding is consistent with Glosten et al. (1993). They report negative estimates of λ for excess returns from the CRSP value-weighted market portfolio. Poon and Taylor (1992) and Taylor (2000) also find insignificant estimates of λ . However, Bollerslev et al. (1992) also state that almost all of the early studies find λ is positive and significantly different from zero at the 5 percent level. Blair et al. (2002) estimate that λ is positive for the daily returns of the S&P 100 index with the null hypothesis $\lambda = 0$ is accepted at the 10 percent level.

Test for Asymmetry

This test shows whether positive and negative innovations affect future volatility differently from the prediction of the model.

We test the null hypothesis $H_0: \alpha = 0$

Against the alternative $H_1: \alpha > 0$

The value of α that maximizes the log likelihood $\log L(\theta)$ is 0 and the maximized $\log L(\theta)$ is 4015.71. Thus the LR test value is 0. Comparing this value with the critical value from χ^2 distribution cannot reject the null hypothesis at 5 and 1 percent significance levels. Accepting the null hypothesis indicates that

the conditional variance is a symmetric function of changes in prices in Dhaka stock market. Increase in price has same impact on future volatility from decrease in price. Alternatively the Wald test can be performed. The Wald test statistic for α is $\frac{\hat{\alpha}}{se(\hat{\alpha})}$. The

parameter estimate of $\hat{\alpha}$ is 0 and the standard error is 0.000949. Thus the Wald statistics for $\hat{\alpha}$ is 0. Comparing this test value with the critical value from the normal distribution cannot clearly reject the null at 5 and 1 percent significance levels.

V. CONCLUSION

Researchers use different methods for volatility forecasting and estimating. Different volatility models perform differently at different times. Empirical findings suggest that GARCH is the most popular structure in estimating and forecasting volatility. Hence, in this paper, we choose two GARCH models and compare these models to see which model performs better in estimating and forecasting volatility. The two GARCH models that we choose are standard GARCH and Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) models. These two models are used for the conditional normal distributions. Daily returns data is used to model the volatility. The DSI index (All Share Price Index) of the Dhaka Stock Exchange is chosen to test the volatility models. We use the maximum likelihood estimation (MLE) technique to estimate the parameters of the chosen models. Results indicate that GJR-GARCH model has higher log-likelihood than GARCH model. This infers that GJR-GARCH model is better performing model to estimate and to

forecast volatility in the Dhaka stock market. Some other hypothesis tests are performed by using the likelihood ratio (LR) and the Wald test. Results from those tests indicate that volatility process has a unit root i.e. volatility process is nonstationary, expected returns do not always depend on volatility, and the conditional variance (volatility) of future asset price is a symmetric function of changes in price at Dhaka stock market.

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